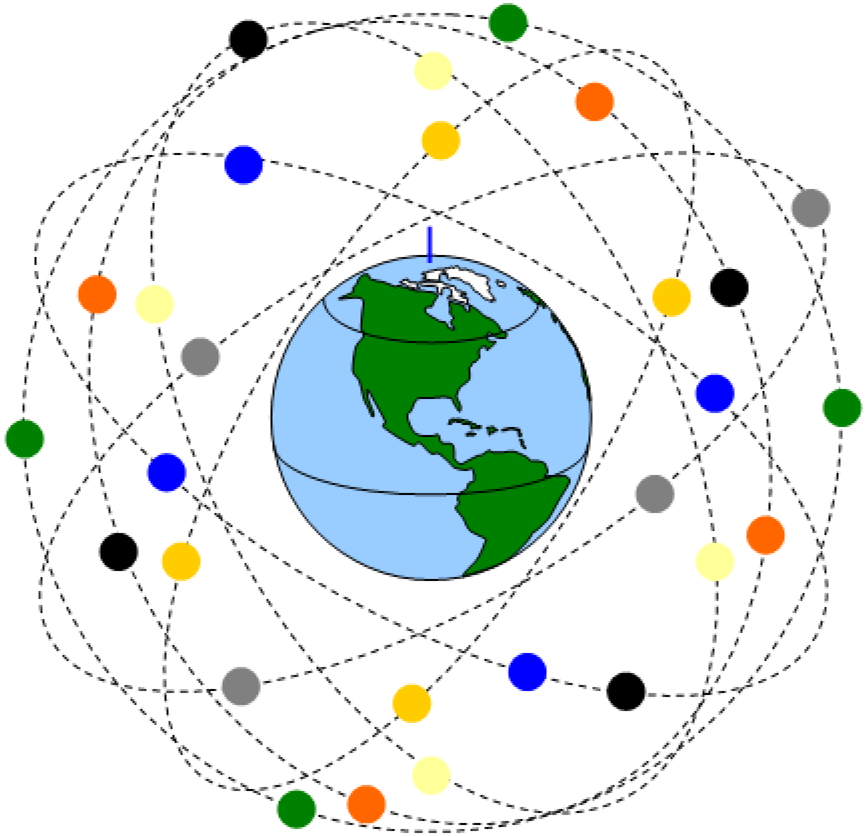
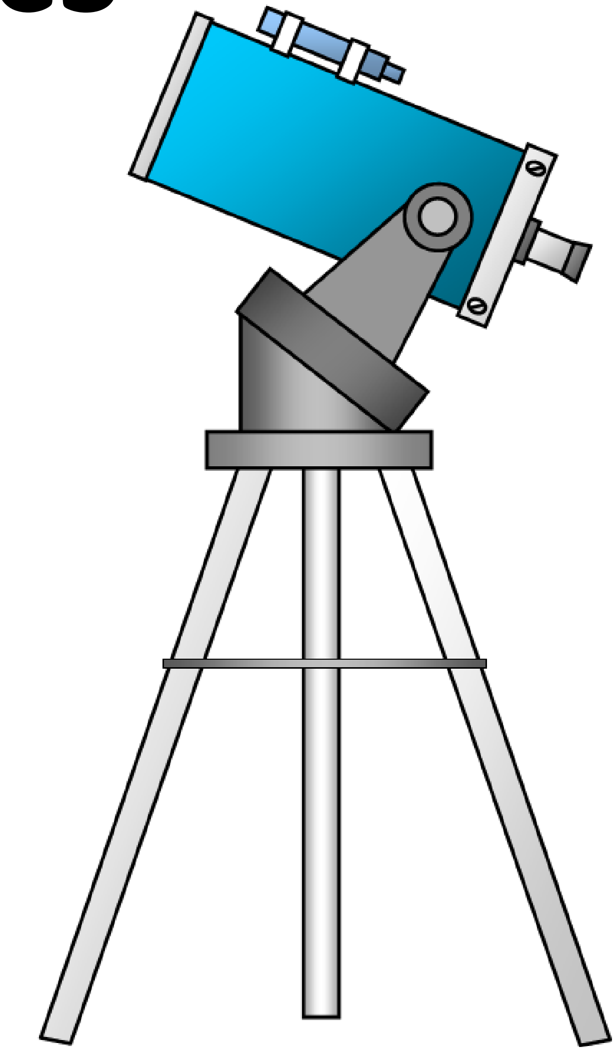
Astrophysics

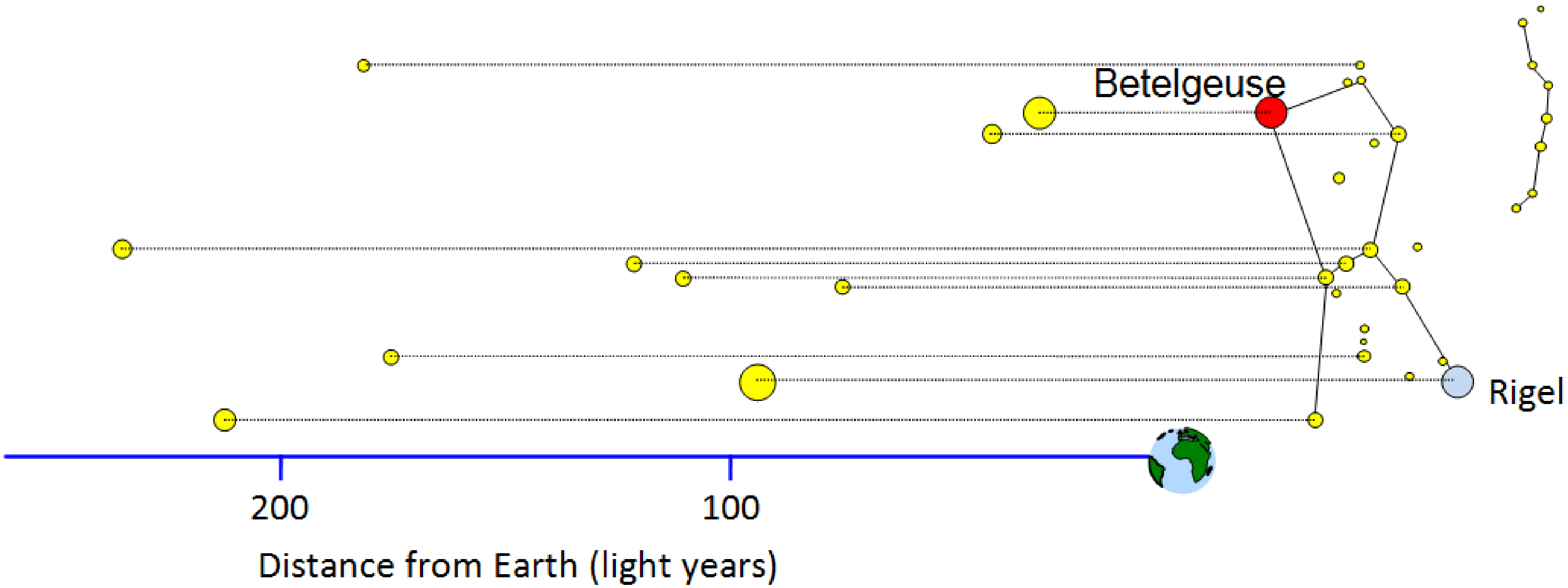




Clusters and constellations

Star clusters are groups of stars that are ‘connected’ by a significant gravitational force ands move around tougher as the galaxy rotates. The motion of the Sun through the galaxy does not affect the appearance of a star cluster from Earth over a long period of time.

A constellation is a group of stars that appear to be related simply because of the view of them from the Earth. They may be at very different distances from the Earth and so as time passes the appearance of the constellation will change.



Solar energy

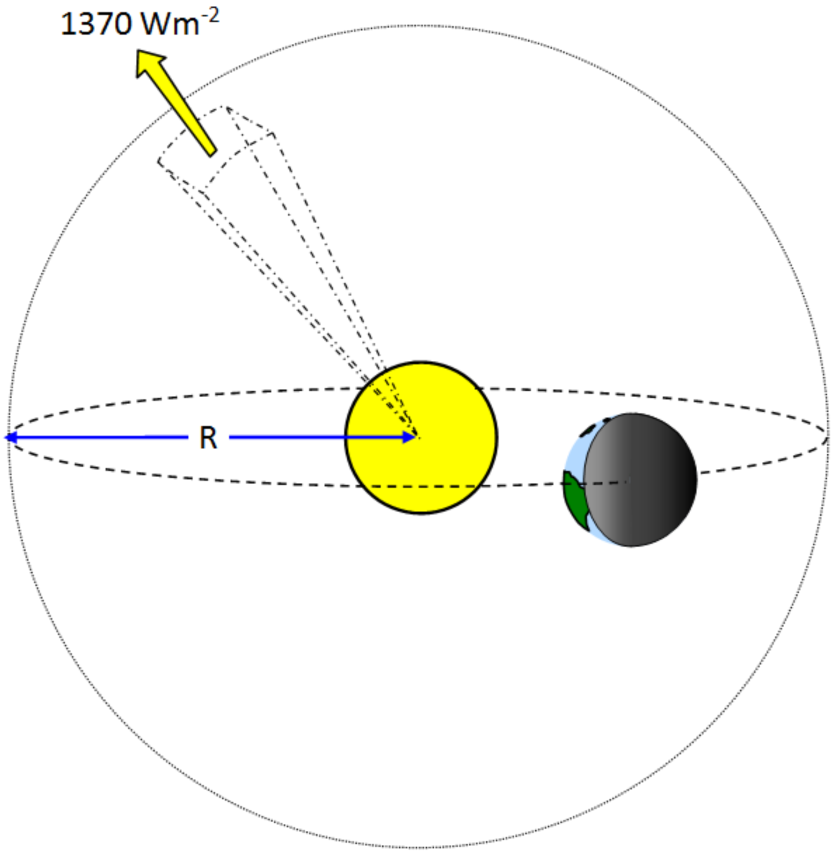
The solar energy falling on every square metre every second at a distance equal to the radius (R) of the Earth's orbit is 1370 Wm^{-2}. This means that a total amount of energy equal to $4\pi R^2$ x1370 J is passing out from the Sun every second.

The radius of the Earth's orbit is 1.5×10^{11} m and so the energy emitted by the Sun every second is:

$E = 4\pi(1.5 \times 10^{11})^2 \times 1370 = 3.87 \times 10^{26}$ J.

Using $E = mc^2$ the mass converted to energy by the Sun every second = E/c^2 = $3.87 \times 10^{26}/9 \times 10^{16} = 4.3 \times 10^9$ kg

This means that the Sun is converting over four million tonnes of its mass into energy every second!



The mass-luminosity relation

For stars in the Main there is a fairly simple relationship between the mass of the star and its luminosity. The more massive the star the more luminous it is.

This can be expressed as a formula called the **mass-luminosity relation.**

Luminosity of star B/Luminosity of star A = [Mass of star B/Mass of star A]$^{3.5}$

This formula is valid only for main sequence stars, not for white dwarfs, red giants or red supergiants and even for the main sequence the masses must lie between 0.08 and 80 solar masses.

One difference between Main Sequence stars and other groups of stars is their density. Main Sequence stars (including our Sun of course) have density around 1600 kgm^{-3} while the density of a red giant is about 10^{-4} kgm^{-3} and that of a red supergiant nearer 10^{-6} kgm^{-3}.

Power output of a star

The Stefan-Boltzmann law states that the power emitted by a black body of surface area A and with a surface temperature T (K) is given by the equation:

Power = σAT^4 where σ is a constant ($5.7\text{x}10^{-8}$ $\text{Wm}^{-2}\text{K}^{-4}$).

(Note: assume that the temperature of the surroundings (deep space) is 0 K)

If we assume that a star is roughly spherical then $A = 4\pi r^2$ for a star of radius r.

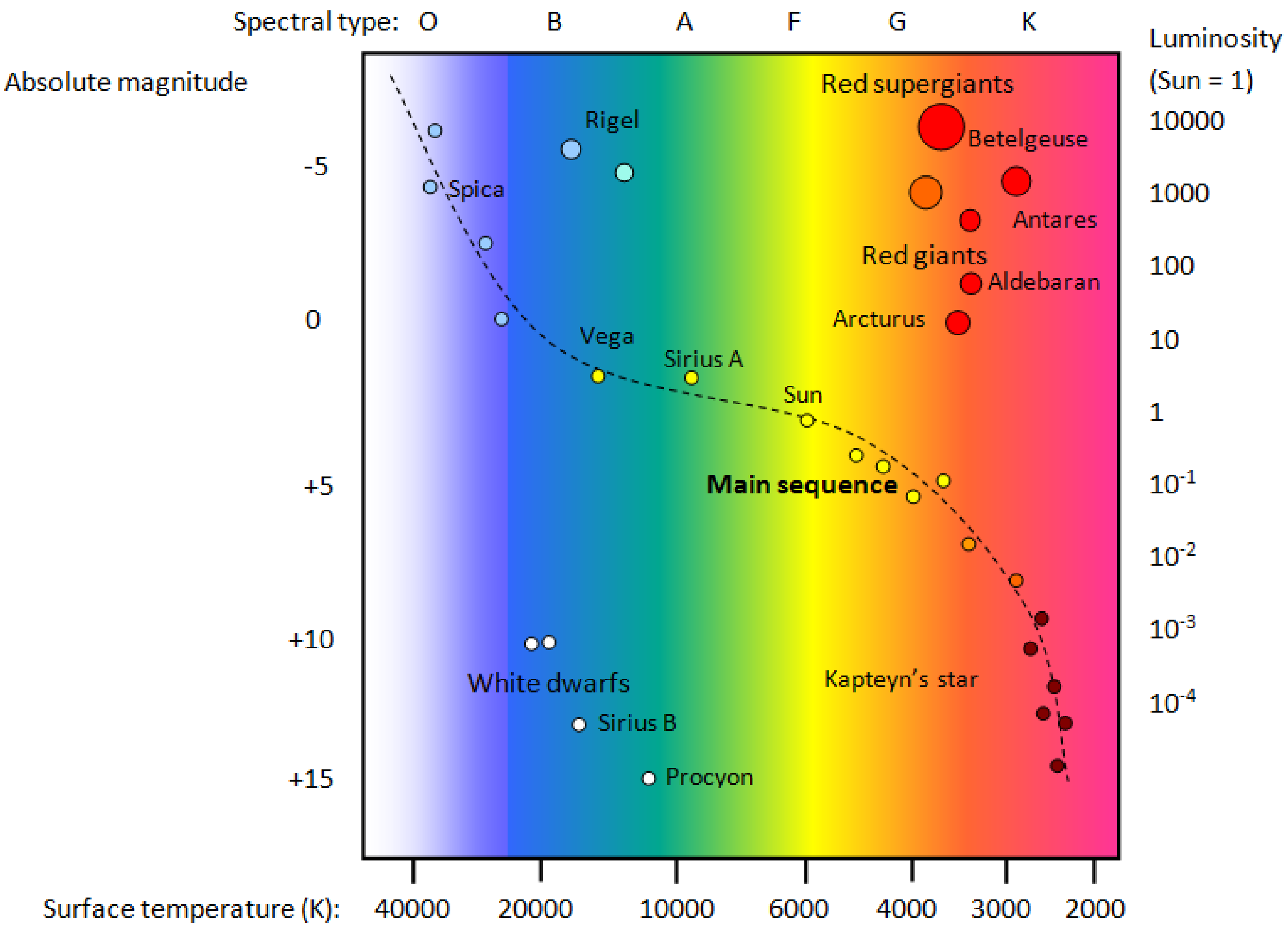
The power of a star is therefore $4\pi\sigma r^2T^4 = 7.16\text{x}10^{-7}r^2T^4$.

Consider our Sun. It is a star of surface temperature 6000 K, and a radius $6.96\text{x}10^8$ m. Using the preceding equation we can calculate its power output:

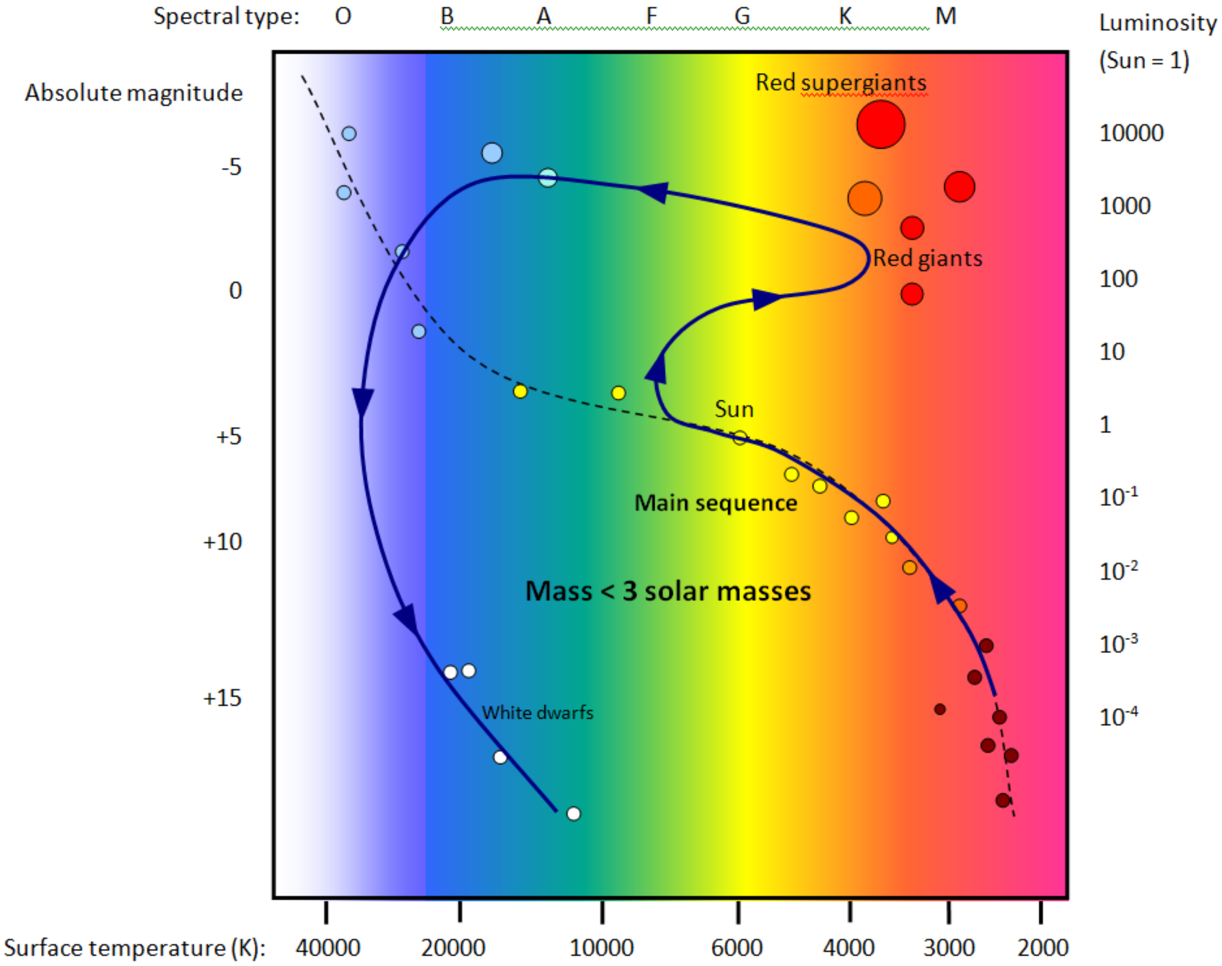
Power output of the Sun $= 7.16\text{x}10^{-7}r^2T^4 = 7.16\text{x}10^{-7}\text{x}[6.96\text{x}10^8]^2\text{x}[6000^4]$

$= 7.16\text{x}10^{-7}\text{x}\ 4.84\text{x}10^{17}\text{x}1.296\text{x}10^{15} = 4.5\text{x}10^{26}$ W

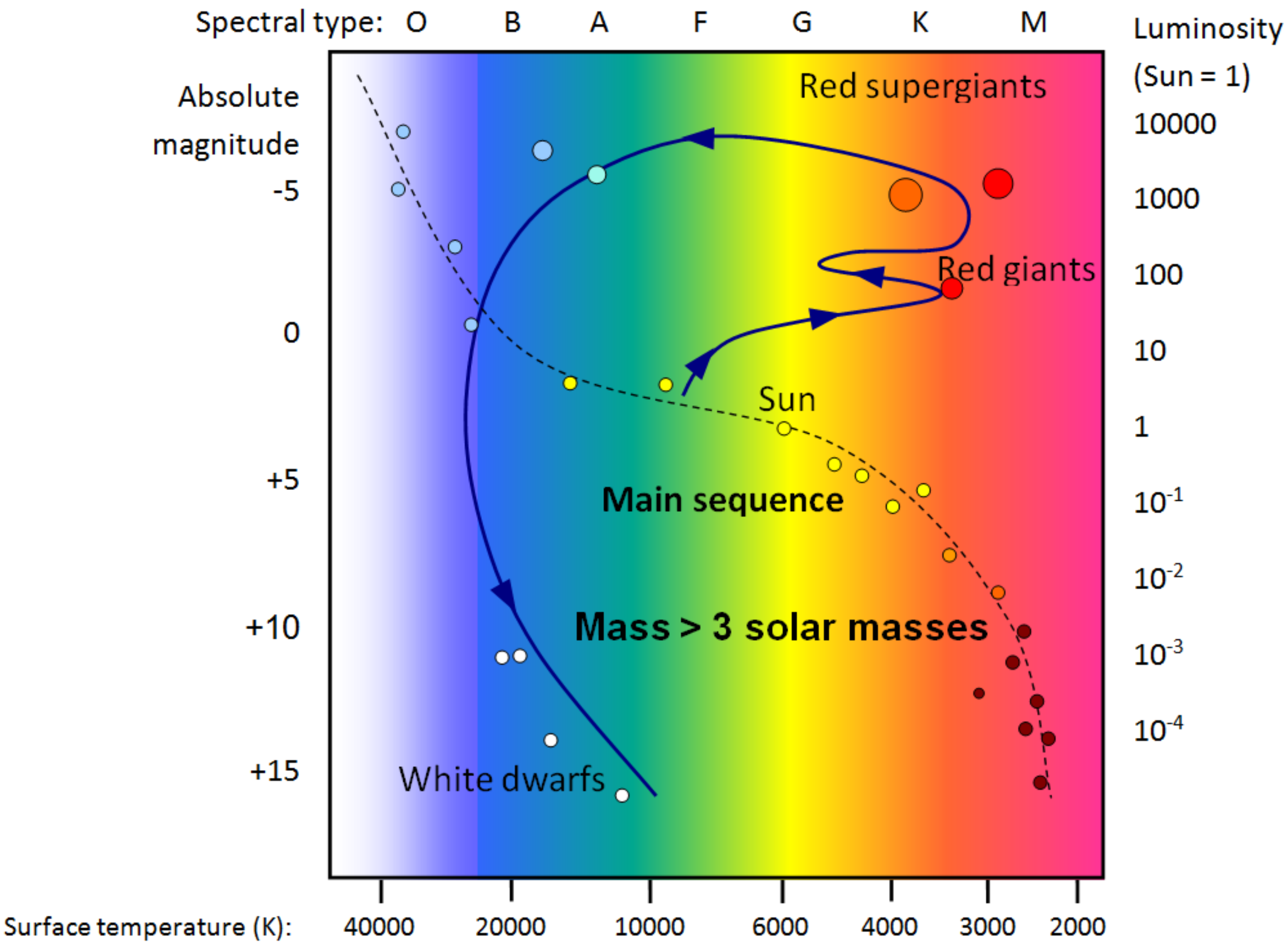
Hertzsprung-Russell diagram



Evolution of stars



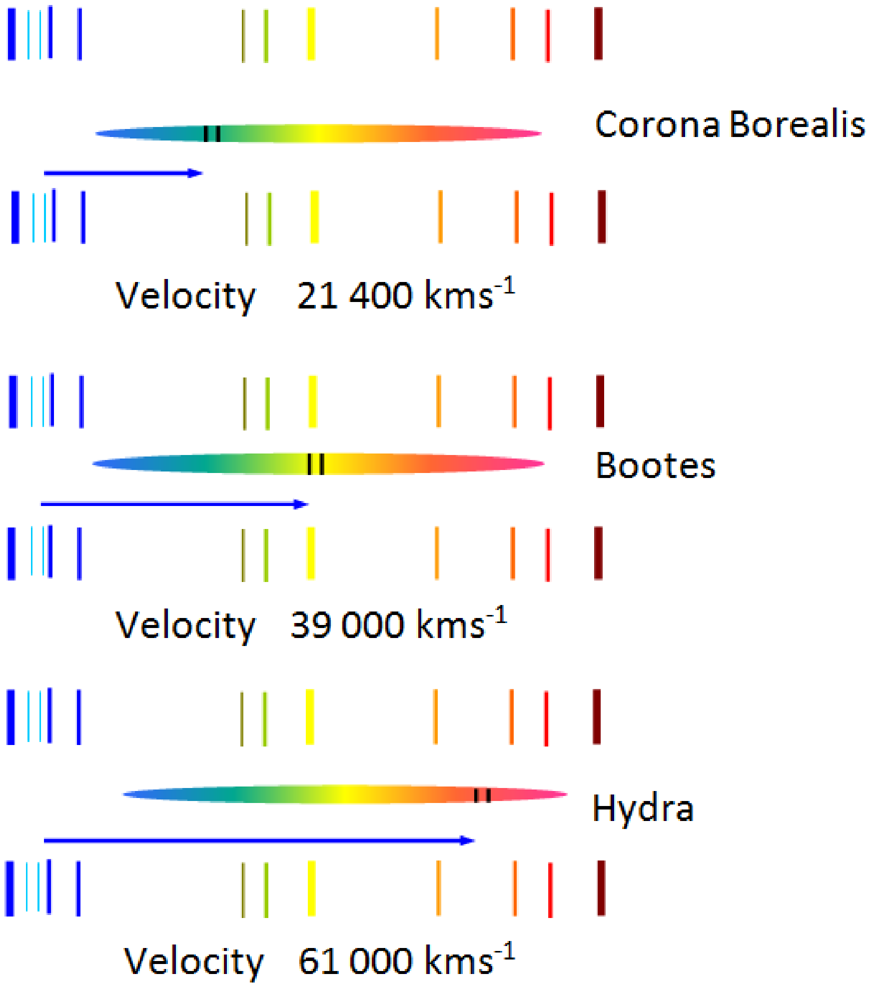
Evolution of stars (M>3M$_{SUN}$)



Galactic red shift

Light coming from distant galaxies is shifted significantly towards the red and this shows that they are moving away from us at high speeds, many tens of thousands of kilometres per second. This shift towards the red is called the **Red Shift** and is very good evidence for the expansion of the Universe and for the origin of the Universe in the Big Bang.

Velocity of galaxy (v) = $\Delta\lambda c/\lambda$ where λ is the wavelength of a line in the same spectrum viewed on Earth, $\Delta\lambda$ is the shift in wavelength and c is the speed of light.



Hubble's Law

The Hubble formula provides a very powerful way of determining not only distances of remote galaxies but also the age of the Universe itself.

Hubble's Law: Velocity of recession (v) = Hubble constant (H) x distance (r)

where r is the distance of the galaxy from the Earth. The value of Hubble's constant is critical to the measurement of the distance of a given galaxy and therefore to the measurement of the size of the Universe. At present its value is thought to be about 70 $\text{kms}^{-1}\ \text{Mpc}^{-1}$.

The velocity of recession of a galaxy increases by 70 kms^{-1} for every 1 Mpc increase in distance.

The value of H can be found by measuring the distance of another galaxy using the period-luminosity relationship for a Cepheid variable star.

Value of H in SI units
Take the Hubble constant H to be 70 $\text{kms}^{-1}\ \text{Mpc}^{-1}$ and one light year to be 9.46×10^{15} m
One Parsec = 3.26 light years = 3.0857×10^{16} m therefore 1 Mpc = 3.0857×10^{22} m
So 70 $\text{kms}^{-1}\text{Mpc}^{-1}$ $= 70\times10^{3} / 3.09\times10^{16}\times10^{6} = 2.27 \times 10^{-18}\ \text{ms}^{-1}\text{m}^{-1}$

Critical density of the universe

The velocity of recession of a galaxy can be considered as its escape velocity from the rest of the Universe

Velocity of recession (v) = HR where R is the distance of the galaxy.

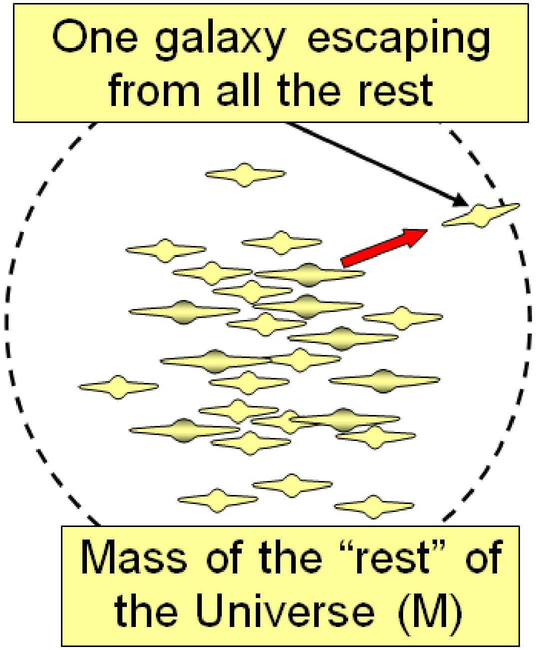
But escape velocity (v) = $\sqrt{[2GM/R]}$
so $v^2 = 2GM/R$

where G is the gravitational constant

Therefore:
$v^2 = 2GM/R = [2G\rho[4/3]\pi R^3]/R = 8/3[G\rho\pi R^2]$

From the first equation $v^2 = H^2R^2$ and so:

$$\boxed{\text{Critical density } (\rho) = 3H^2/8\pi G}$$

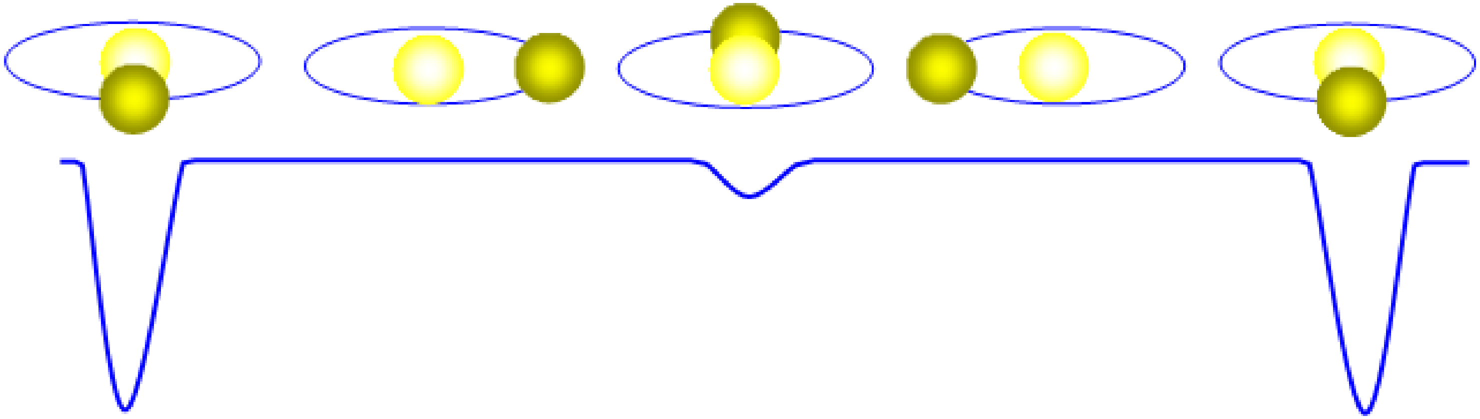


One galaxy escaping from all the rest

Mass of the "rest" of the Universe (M)

Eclipsing variable

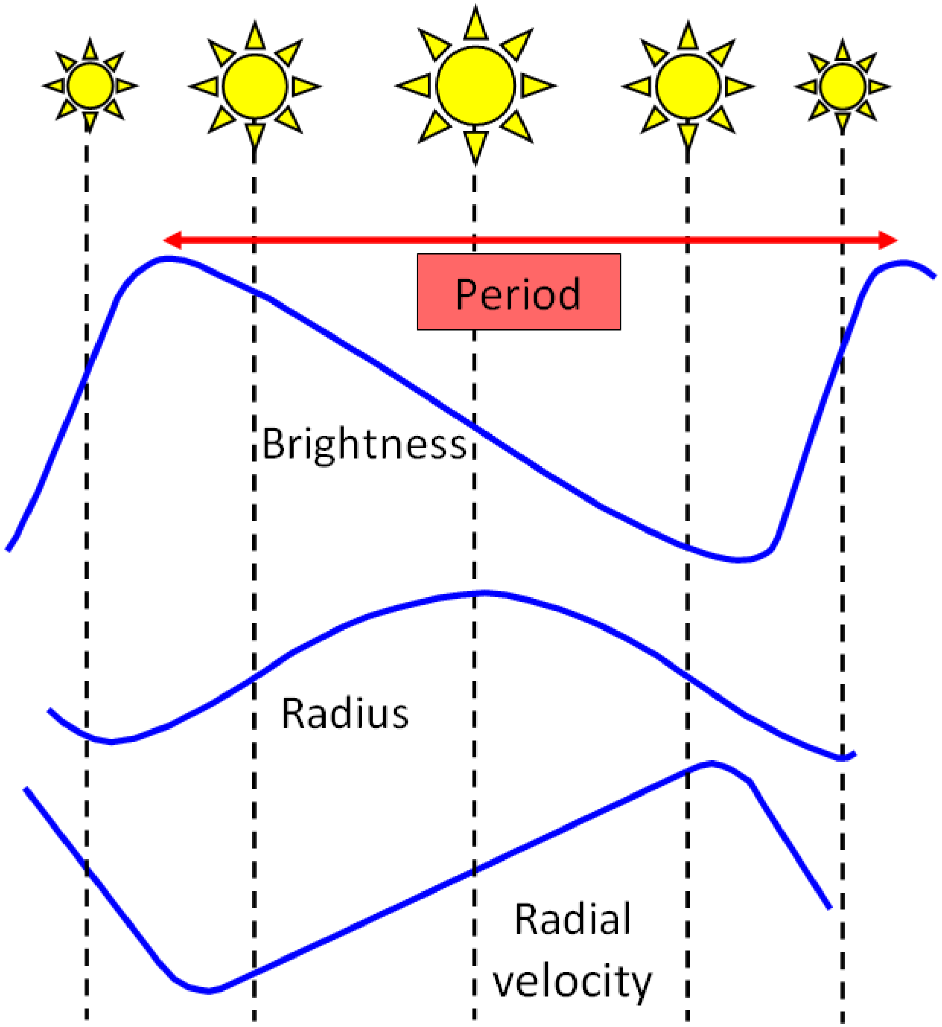
As one star orbits another one or other of the stars may be eclipsed by its companion and this affects the total observed brightness of the pair. The larger dip in the observed brightness-time curve is when the dimmer star moves in front of the brighter one. The smaller dip is where the brighter one moves in front of the dimmer star



Cepheid variables

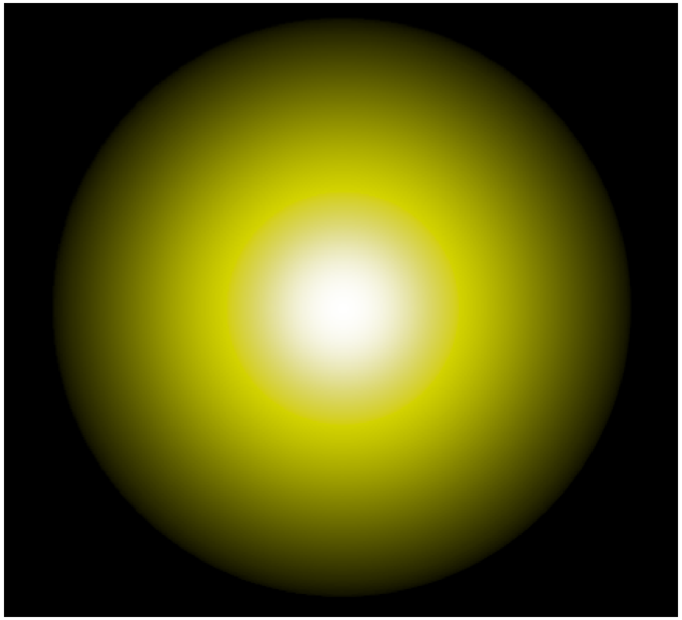
The brightness of a Cepheid variable changes because the size of the star is changing. When the star is small its density is large and the temperature and pressure inside the star increase. This makes the star expand.

When the star is large gravity causes it to contract again and the cycle repeats itself. The longer the period of luminosity variation the more luminous the star. This is known as the **period-luminosity relationship**.



The Big Bang

It is now generally accepted by most astronomers that the Universe as we know it began with an unimaginably huge explosion 13 700 000 000 years (1.37×10^{10} years) ago.

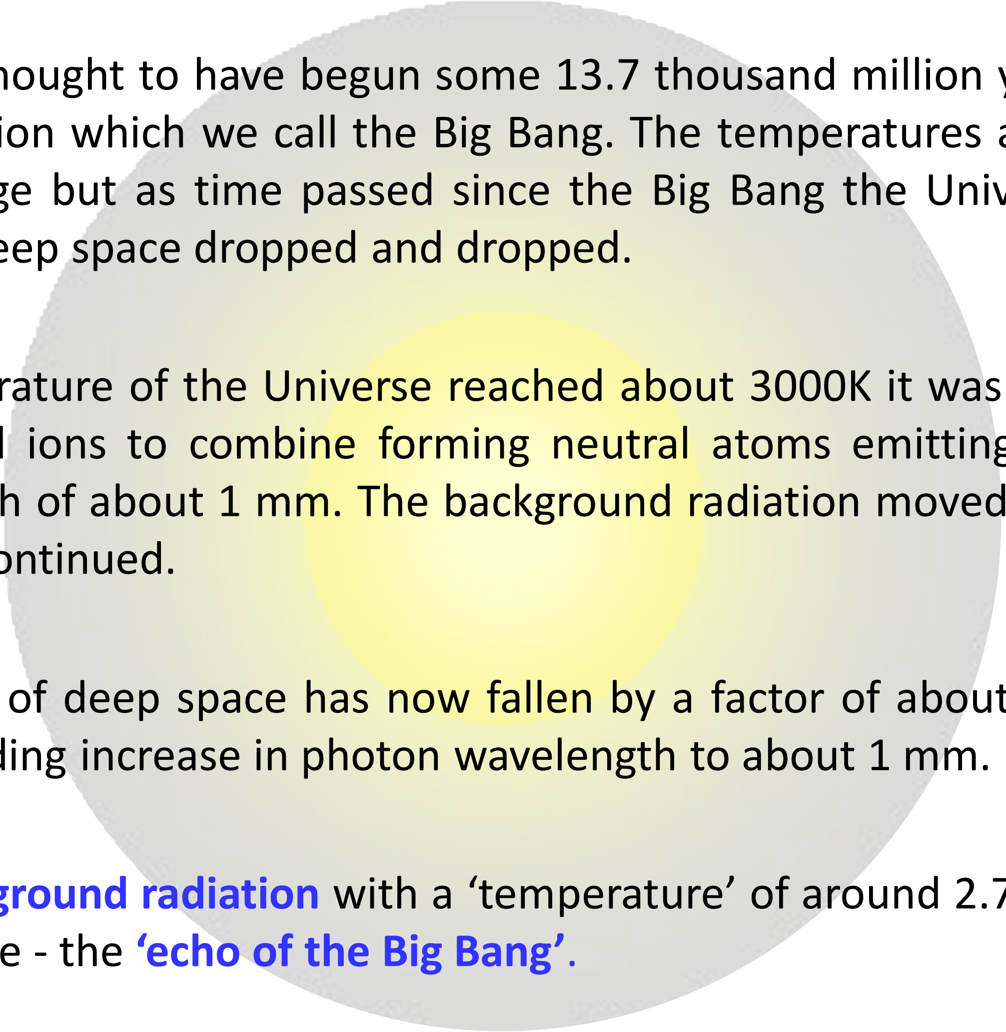


We call this the Big Bang.

The age of the universe is therefore 1.37×10^{10} years.

Time and space both originated at the same time with the Big Bang. Before that there was no space and no time – the Big Bang ‘created’ space and time. We cannot ask what happened before the Big Bang because before that moment nothing existed – no space and no time!

The echo of the Big Bang



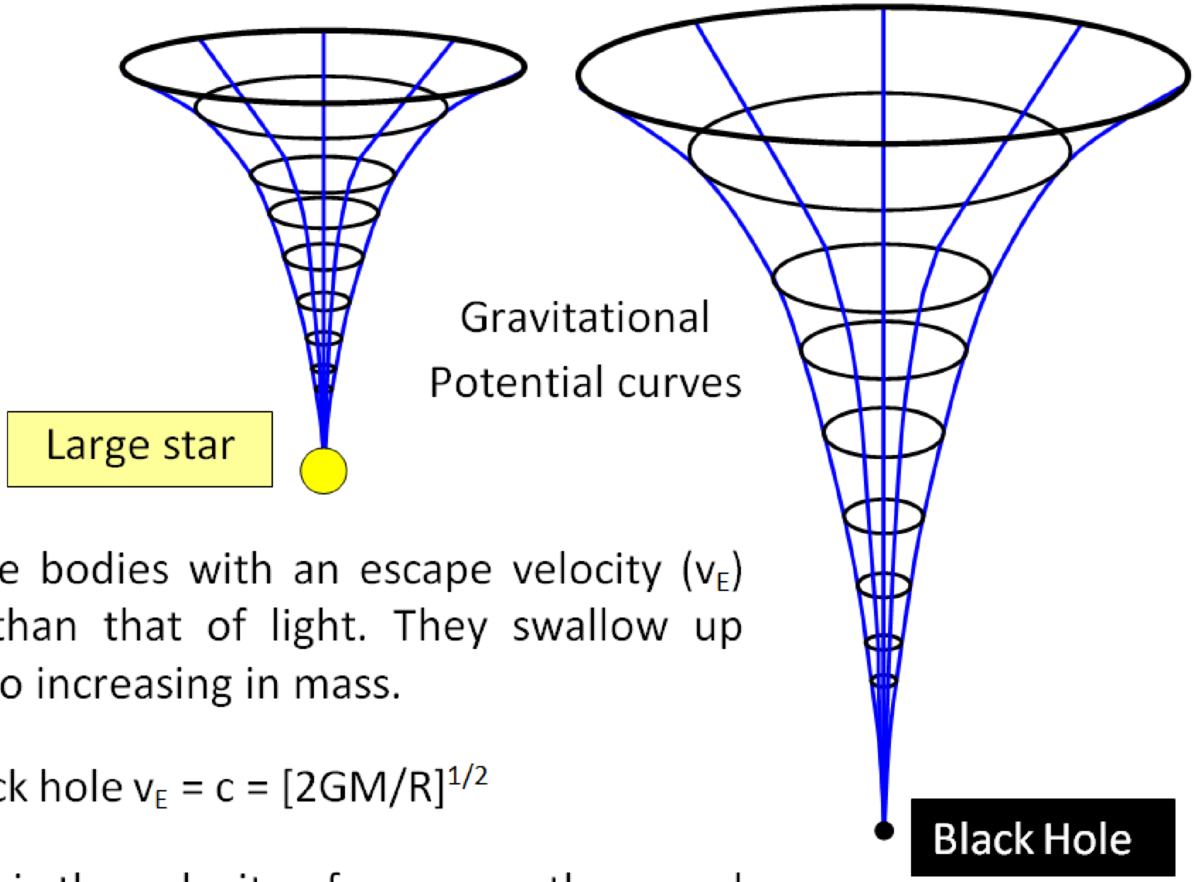
The Universe is thought to have begun some 13.7 thousand million years ago with an enormous explosion which we call the Big Bang. The temperatures at that time were unimaginably huge but as time passed since the Big Bang the Universe cooled. The temperature in deep space dropped and dropped.

When the temperature of the Universe reached about 3000K it was just cool enough for electrons and ions to combine forming neutral atoms emitting photons with a typical wavelength of about 1 mm. The background radiation moved into the infrared and the cooling continued.

The temperature of deep space has now fallen by a factor of about 1000 to 2.725 K with a corresponding increase in photon wavelength to about 1 mm.

This **cosmic background radiation** with a ‘temperature’ of around 2.7K has been given an evocative name - the **‘echo of the Big Bang’**.

Black holes



Large star

Gravitational

Potential curves

Black Hole

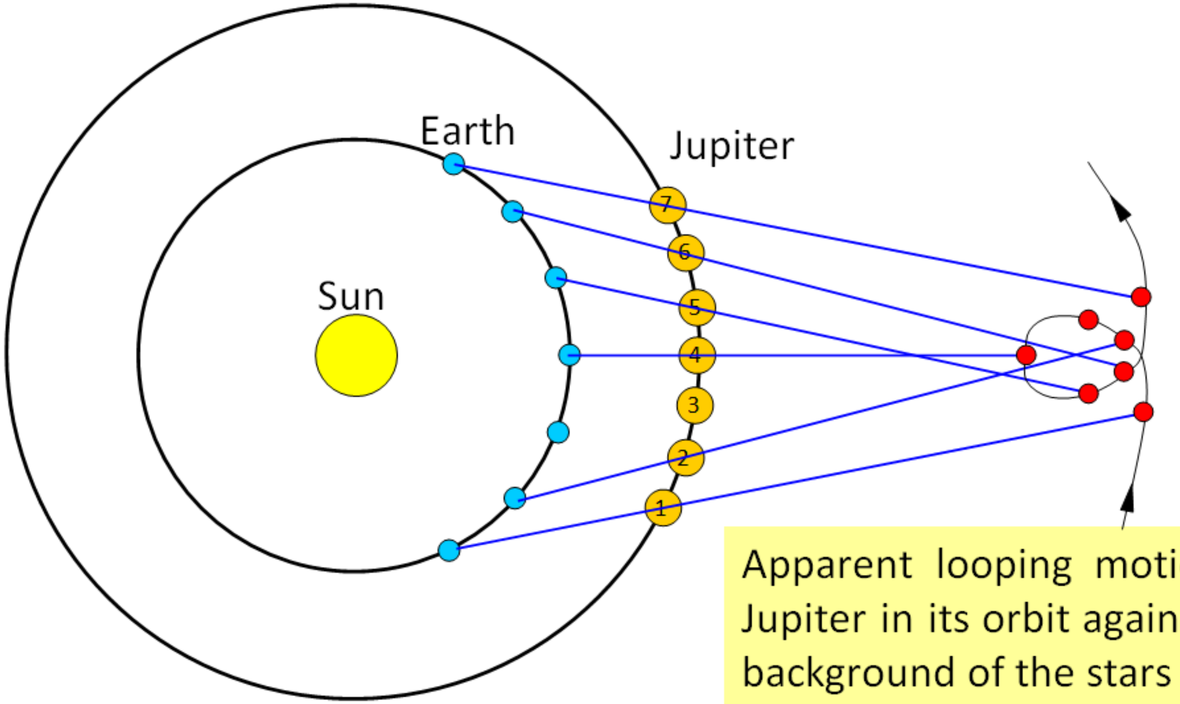
These are bodies with an escape velocity (v_E) greater than that of light. They swallow up matter- so increasing in mass.

For a black hole $v_E = c = [2GM/R]^{1/2}$

where v_E is the velocity of escape, c the speed of light, G the gravitational constant, M the mass of the star and R its radius.

Retrograde motion

If Jupiter (or indeed any of the planets further from the Sun than the Earth) is viewed over a few weeks it shows a looping motion against the distant stars. Indeed there are times when it appears to be moving backwards in its orbit. This apparent backward motion is called **retrograde motion**.



Apparent looping motion of Jupiter in its orbit against the background of the stars

Temperatures of the stars

Assume that stars are perfectly black bodies and then use Wien's displacement to calculate their temperature.

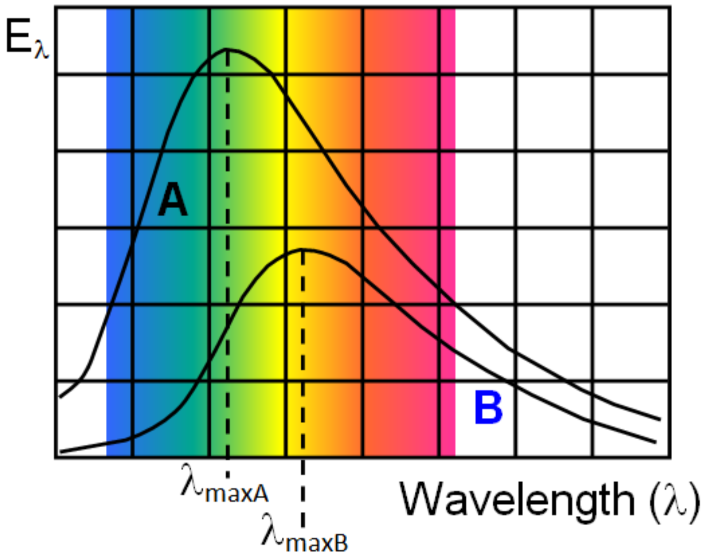
The law states that:

Wavelength (λ_{maxA}) X Temperature (T_A)

= Wavelength (λ_{maxB}) X Temperature (T_B)

where λ_{maxA} ,T_A and λ_{maxB} ,T_B are the peaks of the energy wavelength curve and the absolute temperature of two black bodies, in this case two stars.

Star A has an energy peak nearer the blue end of the spectrum and is at a higher temperature than star B that has an energy peak towards the red end of the spectrum.



Stellar energy

The energy of the stars is produced by nuclear fusion, the temperatures required in the stars being lower than those needed in the laboratory because of the enormous pressures in the stellar interiors.

One such reaction that occurs in the Sun is the so called PP chain shown below.

$$ {}^{1}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \longrightarrow {}^{2}_{1}\mathrm{H} + e^{+} + \text{energy} $$

$$ {}^{2}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \longrightarrow {}^{3}_{2}\mathrm{He} + \text{energy} $$

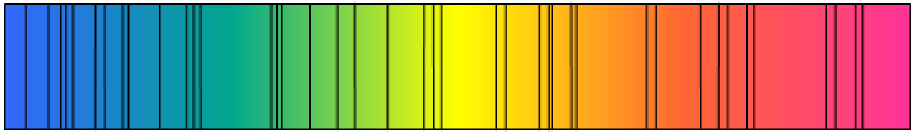
$$ {}^{3}_{2}\mathrm{He} + {}^{3}_{2}\mathrm{He} \longrightarrow {}^{4}_{2}\mathrm{He} + 2\,{}^{1}_{1}\mathrm{H} + \text{energy} $$

Part of the energy in the first reaction is high-speed neutrinos and these can be detected on the Earth.

Heavier and heavier nuclei can be produced by successive fusion reactions.

Stellar spectra

We can in fact find out a surprising amount of information about stars simply by the analysis of their spectra. A simplified version of the type of stellar spectrum that you might observe is shown below.

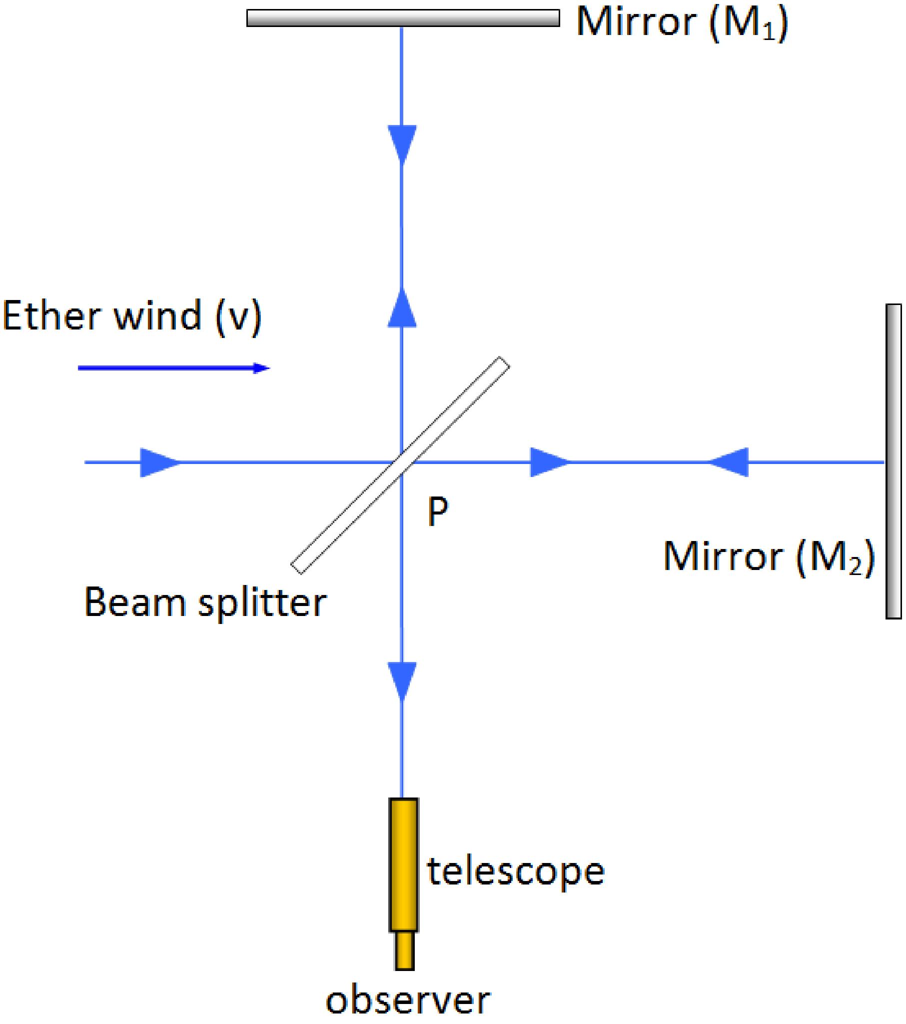


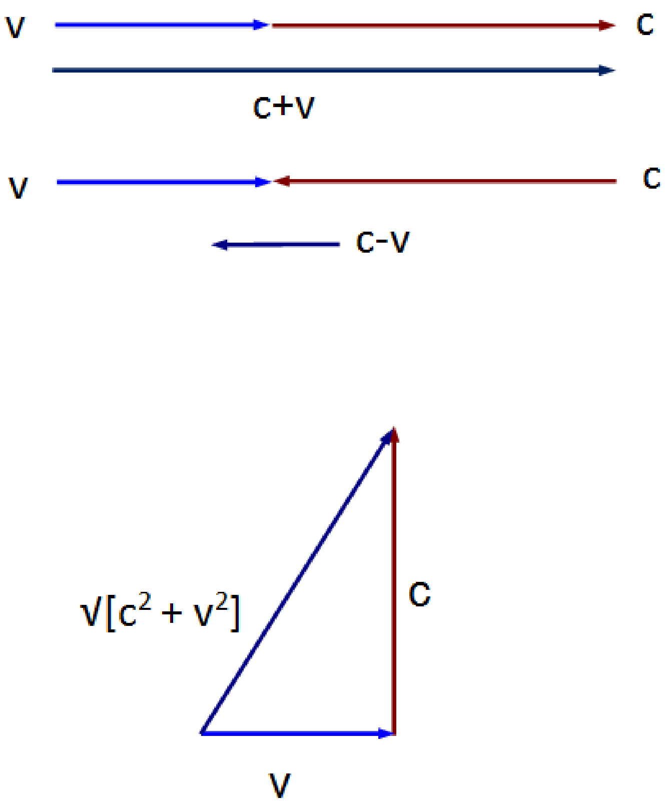
The spectrum is basically a continuous spectrum from violet to red but it is covered with many fine dark lines. These are absorption lines and in a real stellar spectrum there would be many hundreds of these spread across the whole spectrum.

The following table gives a brief description the properties of the main spectral classes.

Spectral class	Surface temperature (K)	Significant features
O	>20 000	Helium ions
B	10 000 – 20 000	Atoms of helium and hydrogen
A	7000 – 10 000	Hydrogen atoms (Balmer series of spectral lines appear). Calcium, magnesium and silicon ions
F	6000 – 7000	Calcium ions. Iron and sodium atoms and ions
G	5000 – 6000	Calcium ions (Ca II) strong. Other metallic lines
K	3500 – 5000	Neutral metal atoms and some molecular bands appearing.
M	2000 - 3500	Molecular bands such as titanium oxide (TiO) and metal atoms.

Michelson-Morley experiment (1)





Michelson-Morley experiment (2)

If the velocity of the ether is v relative to the Earth then this must affect the velocity of light relative to the Earth.

In the down wind direction the velocity of light will be c + v, in the upwind direction c – v.

The time (t_2) taken for the light to travel to the beam splitter and back is:

$t_2 = d/[c+v] + d/[c - v] = 2d/c[1 - d^2/c^2]^{-1}$

If the light beam was shone at right angles it would mean that the Earth would be travelling at right angles to the direction of the ether wind.

The time taken for it to return to the observer would then be:

$t_1 = 2d/\sqrt{[c^2 + v^2]} = 2d/c[1 - d^2/c^2]^{-1/2}$.

The two times (t_1 and t_2) are different and so the light beams should return to the observer with a difference in phase and so the interference pattern should show a shift.

The apparatus was rotated to check the fringe shift for varying directions of the light beam. However no such shift in the interference fringes was observed. The two times measured by the experiment were equal and so the speed of light seemed to be constant no matter in which direction the light beam was travelling relative to the Earth's motion.

This result proved that the ether did not exist.

Apparent and absolute magnitude

How bright a star looks is given by its apparent magnitude (m). This is different from its absolute magnitude. The absolute magnitude (M) of a star is defined as the apparent magnitude that it would have if placed at a distance of 10 parsecs from the Earth.

Consider two stars A and B. Star A appears to be brighter than star B. In other words the intensity of the light reaching the observer from star A is greater than that from star B.

Let the apparent magnitude of star A = m_A and the apparent magnitude of star B be m_B.

A magnitude difference of 5 is a difference in intensity (I) by a factor of 100 and so:

$$\boxed{I_A/I_B = 100^{(m_B - m_A)/5}}$$

Magnitude and intensity

Let the magnitude of star A (m_A) be that at 10 parsecs, in other words the absolute magnitude of the star (M) and let m_B be the magnitude (m) at some other distance d (also measured in parsecs).

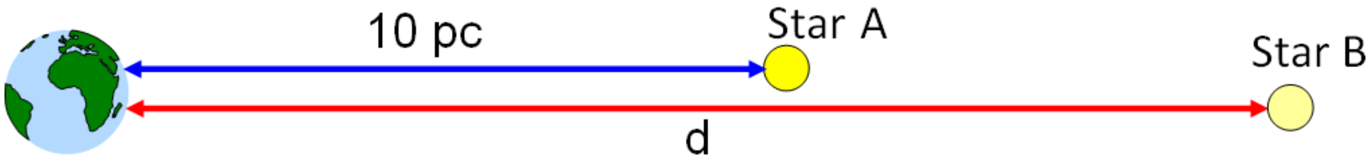
Therefore :

$$m - M = 5/2[\lg(I_A/I_B)]$$

But from the inverse square law: $(I_A/I_B) = (d_B/d_A)^2$ because the intensity is inversely proportional to the square of the distance of the star.

Therefore :

$$\boxed{M = m + 5 - 5\lg(d)}$$



The fate of the Universe

The fate of the Universe depends on how much matter there is.

The Hubble constant H = 70 $\mathrm{kms^{-1}\ Mpc^{-1}}$ gives a critical density for the Universe is $9.2\times10^{-27}\ \mathrm{kgm^{-3}}$. Actual measurements show a density of $4\times10^{-28}\ \mathrm{kg\ m^{-3}}$. This is too low to prevent a run-away expansion but astronomers believe that a large amount of the mass of the Universe exists as "dark matter", much of which is so far undetected.

If W = Actual average density of matter in the Universe/Critical density of the Universe the critical density will decide whether the Universe is:

(a) open – a run-away expansion (W<1)

(b) flat – an expansion but slowing (W = 1)

(c) closed – a final contraction,

the Big Crunch (W>1)