Mechanics problems

Question:

1. A T-1200 engine can propel a snowmobile with a certain force, When the T-1200 engine is installed in my snowmobile, it can accelerate on frictionless ice from 0 to 10 m/s in 5 seconds. When the T-1200 engine is installed in Jen's snowmobile, it accelerates from 0 to 10 m/s in 10 seconds.
In this problem, to keep the maths simple, let's pretend that snowmobile drivers are massless.

(a) With the information given, what can you say about the mass of my snowmobile versus the mass of Jen's snowmobile?

Which one is more massive, and by what ratio (i.e., three times as massive, 1.5 times as massive, or what)? Use your intuition, don't plug in any formulas to solve this part.

(b) My snowmobile, it turns out, has a mass of 200 kg including the engine) What force does the T-1200 engine exert on a Snowmobile?

(c) Suppose my snowmobile breaks down. Jen offers to tow my snowmobile with her snowmobile, by attaching them with a strong cord. Jen's snowmobile has a T-1200 engine.

While she's towing my snowmobile, how fast can she accelerate?

2. A medieval army is attacking a castle with very tall walls, 100 meters high. The army's cannon is entrenched exactly 50 meters from the castle. The Head Knight decides that the cannon can cause the most damage to the castle it cannon balls are fired over the castle wall. Specifically, the head knight wants the cannon ball's trajectory to be such that its peak (i.e. the cannon ball's highest point) is reached when the cannon ball is directly over the wall. The cannon fires balls at 80m/s.
(a) At what angle θ [theta] to the ground should the dead Knight align the cannon?
(b) If the Head Knight screws up and aligns the cannon at $\theta = 60^{o}$ angle to the ground, the cannon ball wall hit the wall.
How high off the ground will the ball hit the wall?

3. A block of mass m_1 = 3.70 kg on a frictionless ramp of angle 28^{o} is connected via a massless cord to a second block, of mass m_2 = 1.86 kg. The cord passes over a pulley which is massless and frictionless. The blocks are released from rest.

(a) what is the magnitude and direction of the acceleration of block 2?

(b) What is the force exerted by the cord on block 1?

4. While driving down the highway at 20 m/s (which is about 72 miles per hour) John spots a police car, sirens flashing. At exactly 1:00 p.m., when the police car is also traveling at 30 m/s and is 100 meters behind John's car, John floors the gas pedal, giving his car an acceleration of 2 m/s^2. John keeps the pedal floored for 5 seconds. During this time, the police car continues moving at 30 m/s, and the officer radios for backup.

(a) How fast is John's car moving at the end of those 5 seconds?

(b) At the end of those 5 seconds, how much distance is between John's car and the police car?

At the end of those 5 seconds, John, in a fit of lawfulness, takes his foot entirely off the pedal. As a result, his car slows down at a rate of 2 m/s^2. At what time will John's car come to a complete halt?

5. A powerful cannon shoots cannon balls at a speed of 100 m/s. When I align the cannon at an angle [theta] to the ground and fire the cannon, the ball takes exactly 1 second to reach a height of 50 meters above the nozzle of the cannon. To simplify calculations, let's say the acceleration due to gravity is 10 m/s^2.

(a) What's theta, in degrees?

(b) One second after it's fired, how far away from the cannon is the ball

(c) How fast is the cannon ball moving one second after it's fired?

6. A ball on a frictionless frozen pond is sliding, northward, at a speed of 20 m/s. At time t = 0, an easterly wind starts to blow, and keeps blowing until time t = 5 seconds
Between t = 0 and t = 5, the wind caused the ball to accelerate. The eastward acceleration, in meters per second per second, is given by the formula, a = 10 - 2t
At time t = 5 seconds, what is the ball's speed, and in what direction is it going?

7. A physics teacher wants to demonstrate constantly-accelerated motion by letting a (frictionless) ice cube slide down an inclined plane. The ramp is t = 1.5 meters long. The teacher can control the angle [theta] that the plane makes with the floor. The teacher plans to release the ice cube from rest at the top of the ramp. In order for the students to see everything clearly, the teacher wants the ice cube to take exactly t = 3
seconds to slide all the way down the ramp.

What angle [theta] should the ramp make with the floor?

6. A worker drags a crate across a factory floor by pulling on a rope tied to the crate. The worker exerts a force of 450 N on the rope, which is inclined at a 38^o angle to the floor. The floor exerts a horizontal resistive (i.e., 'backwards") force of F =125 N.

Calculate the acceleration of the crate.

Answers:

1. Acceleration of my snowmobile = 10/5 = 2 m/s^2
Acceleration of Jen's snowmobile = 10/10 = 1 m/s^2
(a) The forces on both snowmobiles are the same (same engine) so: F= ma
Mass of my snowmobile (m) x its acceleration = mass of Jen's snowmobile (M) x its acceleration

mx2 = Mx1 so M = 2m

(b) Force = ma = 200 x 2 = 400 N
(c) Total mass = 200 + (2x200) = 600 kg acceleration = F/total mass = 400/600 = 2/3 m/s^2

2. (a) Vertically at the top v = 0 and using $v^2 = u^2 + 2as$ with a = -g = - 9.8 m/s^2
u = 80 sinA
$(80\sin A)^2 = 2 \times 9.8 \times 100 = 1960$
this gives A = 33.6 degrees
(b) For theta (I have called it A to make the typing simpler) = 60 degrees.

Horizontally the ball goes 50 m before it hits the wall.
Horizontal velocity = 80 cos 60 and so using s = vt: 50 = [80cos60]t gives t = 1.25 s
Now consider the vertical motion:
Use: $h = s = ut + ½ at^2$ with a = g = - 9.8.
(The reason for the negative sign is that g acts downwards with an initial velocity of the ball directed upwards).
$h = 80\sin 60 \times 1.25 – ½ \times 9.8 \times 1.25^2 = 86.6 – 7.66 = 78.9$ m

3. Can I do part (b) first.
(b) Net force in the cord = 1.86x9.8 – 3.7x9.8xsin28 = 1.21 N
(a) Acceleration of **BOTH** blocks = 1.21/(total mass) = 1.21/(1.86 + 3.7) = 0.22 m/s^2
(they are fixed together by the cord)

4.(a) v = u + at = 30 + 2x5 = 40 m/s
(b) Relative to the police car the initial velocity of John is zero. So we will find out his relative increase in separation after the five seconds of acceleration
This is: $s = ½ at^2 = 0.5 \times 2 \times 25 = 25$ m
Therefore actual separation of the two cars is [100 + 25] = 125 m
(c) Time to decelerate = v/a = 40/2 = 20 s
Now time of acceleration = 5 s   Time of deceleration = 20 s
Therefore John's car will come to a complete halt at 1:00:25

5. Taking g = 10 m/s^2 Again I will use A for the angle.
(a) Use: $h = ut + ½ gt^2$ with g = -10 m/s^2 in the equation. (For reason see above) t = 1 s.
Vertical motion:
$50 = [100 \sin A]\times 1 – 0.5 \times 10 \times 1^2$ this gives 45/100 = sin A   A = 26.7 degrees
(b) Horizontally s = vcosA x t = 100 x cos 26.7 = 89.3 m
(c) At this moment vertical velocity (100sinA – 10x1) = 34.9 m/s
The horizontal velocity is constant at 100 cos A = 89.3 m/s so
Final velocity = square root $(34.9^2 + 89.3^2)$ = 95.9 m/s

6. The acceleration is not uniform and the equation: a = (10 – 2t) needs rewriting as:

dv/dt = 10 – 2t or dv = (10 – 2t) dt

this can be integrated to give $v = 10t – 2t^2$

At the moment you need t = 5 s and so v here is 50 – 25 = 25 m/s to the east.
Now combining this (by Pythagoras) with the northerly velocity we have the final velocity:

v = square root (400 + 625) = 32 m/s.

Direction is tan A = 20/32 = 0.625 gives A = 32 degrees with the west-east line.

7. Acceleration down the slope = g x sin A
Use: $s = ut + ½ at^2$ with u = 0.

$1.5 = ½ \times 9.8 \times \sin A \times 3^2$ so   sinA = 1/[9.8x3]   Therefore: A = 1.95 degrees.

8. I don't think that you have enough information here. You can get the force on the crate but without the mass it is not possible to find the acceleration.

Anyway, the net force is: 450 cos38 – 125 = 355 – 125 = 230 N.

The acceleration is then simply 230/M where M is the mass of the crate.