Mechanics questions 2

1. A person standing on a cliff drops a feather from top of his head. Assume by the time it clears the edge of the cliff on its way down it has already reached its terminal velocity and therefore descends down with a constant velocity of 3.2 m/s. 10.0 s after the feather passed the edge of the cliff, a small steel ball is dropped from the edge. Assuming that the air drag is negligible for the ball, how long does it take for the steel ball to reach the feather?
(Ans: 2.9 s)

2. A projectile is shot at a speed of 25.0 m/s from top of a building into the air at an angle of 15 degrees above the horizon. The projectile lands on the flat surface below the building, 100.0 m away measured in horizontal direction) from the release point. Determine a) the time of flight of the projectile, and b) the height of the building.
Ans: 57.2 m)

3. Vector A= 3.i 2.j + a-k, has a magnitude of 6.0. what is the value of a. Vector B= 4.i - b.j + 2.k, makes an angle of 45 degrees with A. What is the value of b. Write down A x B.
(Ans: a = +- 4.8, taking +4.8 only gives b = +7.2 or -1.05, taking +7.2 only gives AxB = 30.6i + 13.2j - 13.6k. These are partial answers as the valid negative solutions are not worked out.)

4. If you are somewhere on the equator, how much is your acceleration due to rotation of Earth. (the radius of Earth is 6400 km) (Ans: 3.4×10^{-2} m/s^2.)

5. Jan (65.0 Kg) arid Howard (82.0 Kg) are holding to the ends of a rope 22.0 m long on the icy surface of a frozen lake with no friction. If Howard pulls the rope with a sustained force of 30.0 N, (a) calculate the acceleration of Jan. (b) What is the acceleration of Howard. (c) How far does each person slide before they meet?
(Ans: a_j = 0.461 m/s^2, a_h = 0.366 m/s^2, x_j = 12.3 m, x_h = 9.7 m)

Answers:

1. Take the height fallen from the cliff as h in a time t for the ball and (t+10) for the feather:

Feather $h = ut = 3.2 \times (t+10)$
Ball: $h = \frac{1}{2} gt^2 = 4.9\, t^2$ (I am taking g = 9.8 m/s^2 throughout)

Therefore: $3.2 \times (t+10) = 4.9\, t^2$

which when rearranged gives: $4.9t^2 - 3.2t - 32 = 0$.
This can be solved by the usual formula to give t = 2.902 = 2.9 s

2. Horizontal motion:
Horizontal velocity is constant and so: $100 = 25t \cos 15$ where t is the time to reach the ground.
This can be solved to give t = 4.14 s

Vertical motion: $-h = 25t \sin 15 - \frac{1}{2} gt^2$ where h is the height of the building
Notice the minus signs here. I am considering upwards as positive.

Substituting for t = 4.14 s gives $-h = 26.79 - 83.98$ and so h = 57.2 m

3. $\underline{A} = 3\underline{i} - 2\underline{j} + a\underline{k}$ and this has a magnitude of 6
$9 + 4 + a^2 = 36$ (by Pythagoras)

$a^2 = 23$ $a = \sqrt{(23)} = 4.8$

$\underline{B} = 4\underline{i} - b\underline{j} + 2\underline{k}$ Magnitude = $\sqrt{(20 + b^2)}$

$\underline{A}.\underline{B} = AB\cos 45$

$\underline{i}.\underline{j} = \underline{i}.\underline{k} = \underline{j}.\underline{k} = 0$ $\underline{i}^2 = \underline{j}^2 = \underline{k}^2 = 1$

$12 + 2b + 2[\sqrt{(23)}] = 6[\sqrt{(20 + b^2)}]/\sqrt{(2)}$

Rearranging this gives the quadratic equation:
$3.5b^2 - 2(6+\sqrt{(23)})b + 31 - 12[\sqrt{(23)}] = 0$
$3.5b^2 - 21.59b - 26.55 = 0$

This is solved by the usual equation $[-b \pm \sqrt{(b^2 - 4ac)}]/2a$ to give

b = 7.22 or – 1.05.

$\underline{A}x\underline{B} = (3\underline{i} - 2\underline{j} + 4.8\underline{k})x(4\underline{i} - 7.2\underline{j} + 2\underline{k})$

$\underline{i}x\underline{i} = \underline{j}x\underline{j} = \underline{k}x\underline{k} = 0$

$\underline{i}x\underline{j} = \underline{k}$ therefore: $\underline{j}x\underline{i} = -\underline{k}$
$\underline{j}x\underline{k} = \underline{i}$ therefore: $\underline{k}x\underline{j} = -\underline{i}$
$\underline{k}x\underline{i} = \underline{j}$ therefore: $\underline{i}x\underline{k} = -\underline{j}$

Therefore:

$\underline{A}x\underline{B} = \underline{i}(-2x2 - 4.8x(-7.2)) + \underline{j}((-3)x2 + 4.8x4) + \underline{k}(3x(-7.2) - (-2)x4) = 30.6\underline{i} + 13.2\underline{j} - 13.6\underline{k}$

4. Centripetal acceleration due to the rotation of the Earth:

$a = v^2/R$ where v is the velocity of the point on the equator and R is the radius of the Earth.
Using T = one day = 86400 s and $v = 2(\pi)R/T$ we have:

acceleration = 0.038 m/s^2

5. The force is applied to each person and so:
Acceleration of Jan = F/Jan's mass = 30/65 = 0.461 m/s^2
Acceleration of Howard = F/Howard's mass = 30/82 = 0.366 m/s^2

Distance travelled = 22m = distance travelled by Jan + distance travelled by Howard

But : $s = ½ at^2$ for each and so:
Jan: $s_J = ½\ 0.461t^2$
Howard: $s_H = ½\ 0.366t^2$
Notice that the time of travel is the same. They will both be moving for the same time before the collide.
So: $s_J + s_H = 22 = ½\ 0.461t^2 + ½\ 0.366t^2 = 0.414t^2$ and so t = 7.29 s

Substituting this value back into the distance equations gives:
Jan $s_J = 12.3$ m and $s_H = 9.72$ m